On a new, more parsimonious, general theory of relativity

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The Core theory of physics

Path integral formulation: 3 dimensionful constants

$$Z_{\rm C} = \int [\mathcal{D}g] \prod_i [\mathcal{D}f_i] \exp\left[\frac{i}{\boldsymbol{\hbar c}} \int d_g^4x \left(\frac{\boldsymbol{c}^4}{16\pi \boldsymbol{G}} R(g) + \mathcal{L}_m^{SM}(f,g)\right)\right], \tag{1}$$

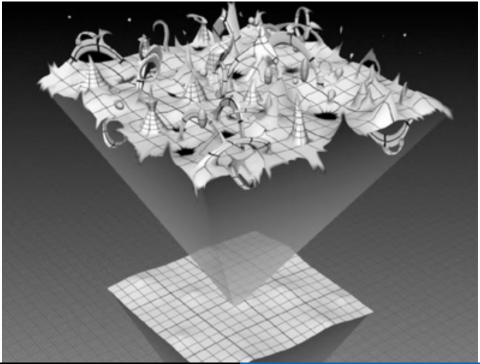
d_g^4x: infinitesimal spacetime 4-volume
$\mathcal{L}_m^{SM}$: standard model Lagrangian density
f_i: standard model fields: gauge bosons, fermions & Higgs
g: spacetime metric
R: Ricci scalar
$\boldsymbol{G}$: Newton's constant
$\boldsymbol{\hbar}$: Planck's quantum of action
$\boldsymbol{c}$: causal structure constant (aka speed of light)

The Core theory of physics

Planck **energy**, Planck **mass**, Planck **time** and Planck **length**:

$$E_P = \sqrt{\frac{\hbar c^5}{G}}, m_P = \frac{E_P}{c^2}, t_P = \sqrt{\frac{\hbar G}{c^5}}, l_P = c t_P. \quad (2)$$

$$+ \text{ Uncertainty principle: } \boxed{\Delta E \Delta t \geq \hbar/2} \quad (3)$$



Entangled relativity

Path integral formulation: 2 dimensionful constants

$$Z_{\rm ER} = \int [\mathcal{D}g] \prod_i [\mathcal{D}f_i] \exp\left[-\frac{i}{2\epsilon^2} \int d_g^4 x \frac{\mathcal{L}_m^2(f,g)}{R(g)}\right], \qquad (4)$$

2103.05313 & 2206.03824

d_g^4x: infinitesimal spacetime 4-volume
$\mathcal{L}_m$: matter Lagrangian density
f_i: fields, such as gauge bosons, fermions and the Higgs
g: spacetime metric
R: Ricci scalar
ϵ: quantum of energy
c: causal structure constant

Entangled relativity

Path integral formulation

$$Z_{\rm ER} = \int [\mathcal{D}g] \prod_i [\mathcal{D}f_i] \exp\left[-\frac{i}{2\epsilon^2}\int d_g^4x \frac{\mathcal{L}_m^2(f,g)}{R(g)}\right] \quad (5)$$

To recover standard QFT ϵ has to be the Planck energy

Check out Moriond proceedings: 2304.09482

Spacetime might not be doomed after all

There are only two universal constants in the definition of the theory:

- **The causal structure constant:** c
- **The Planck energy:** ϵ, whose value is deduced from the $\kappa \approx$ constant limit.

$\Rightarrow$ There is no **Planck length** nor **Planck time** in ER.

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Classical limit: gravity

Classical equivalence (provided $\mathcal{L}_m \neq \emptyset$)

$$-\frac{1}{2}\int \mathrm{d}_g^4 x \frac{\mathcal{L}_m^2}{R} \equiv \int \mathrm{d}_g^4 x \frac{1}{\kappa}\left(\frac{R}{2\kappa} + \mathcal{L}_m\right) \qquad (6)$$

κ: dimensionfull scalar field ($:= 8\pi G/c^4$)

Classical limit: gravity

Classical equivalence (provided $\mathcal{L}_m \neq \emptyset$)

$$-\frac{1}{2}\int \mathrm{d}^4_g x \frac{\mathcal{L}_m^2}{R} \equiv \int \mathrm{d}^4_g x \frac{1}{\kappa}\left(\frac{R}{2\kappa} + \mathcal{L}_m\right) \tag{7}$$

κ: dimensionfull scalar field $\Uparrow$ Cauchy well-posed $\Uparrow$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu} + \kappa^2 \left[\nabla_\mu \nabla_\nu - g_{\mu\nu}\Box\right] \frac{1}{\kappa^2} \tag{8}$$

$$\nabla_\sigma \left(\kappa^{-1} T^{\alpha\sigma}\right) = \mathcal{L}_m \nabla^\alpha \kappa^{-1} \tag{9}$$

Trace metric field equation: $$3\kappa^2 \Box \frac{1}{\kappa^2} = \kappa \left(T - \mathcal{L}_m\right) \tag{10}$$

Solar system phenomenology

Assuming rest-mass is conserved, one has 1308.2770

$$\kappa^2 \Box \kappa^{-2} = \kappa P, \tag{11}$$

where P is the pressure of the considered fluid that source the gravitationnal field equations. The c^{-4} metric of entangled relativity therefore reads

$$\boxed{g_{\alpha\beta} = g_{\alpha\beta}^{GR} + \delta_{\alpha\beta}^{00} \frac{2\delta w}{c^4} + O(c^{-5})}, \tag{12}$$

where $g_{\alpha\beta}^{GR}$ is the solution of general relativity

$$\boxed{\delta w = -G \int \frac{P(\mathrm{x}') d^3 x'}{|\mathrm{x} - \mathrm{x}'|} + O(c^{-2})}. \tag{13}$$

Solar system phenomenology

$$g_{00} = -1 + 2\frac{w}{c^2} - 2\frac{w^2}{c^4} + \mathcal{O}(c^{-5}), \quad \text{(A2a)}$$
$$g_{0i} = -4\frac{w_i}{c^3} + \mathcal{O}(c^{-5}), \quad \text{(A2b)}$$
$$g_{ij} = \delta_{ij}\left(1 + 2\frac{w}{c^2} + 2\frac{w^2}{c^4}\right) + 4\frac{\tau_{ij}}{c^4} + \mathcal{O}(c^{-5}) \quad \text{(A2c)}$$

The equations on the potentials w, w_i and τ_{ij} in entangled relativity are

$$\Box_m w + c^{-2} 4\partial_t J = -4\pi G\left(\sigma - c^{-2}P\right) + \mathcal{O}(c^{-3}) \quad \text{(A3)}$$
$$\triangle w_i - \partial_i J = -4\pi G\sigma^i + \mathcal{O}(c^{-1}), \quad \text{(A4)}$$
$$\triangle \tau_{ij} + \partial_i w \partial_j w - \partial_i J_j - \partial_j J_i - 2\delta_{ij}\partial_t J = -4\pi G\left(\sigma^{ij} + \delta_{ij}P/2\right) + \mathcal{O}(c^{-1}), \quad \text{(A5)}$$

where the source terms are defined as follows

$$\sigma := c^{-2}(T^{00} + T^{kk}) \quad \text{(A6)}$$
$$\sigma^i = c^{-1}T^{0i} \quad \text{(A7)}$$
$$\sigma^{ij} = T^{ij} - \delta^{ij}T^{kk} \quad \text{(A8)}$$

and where one has defined the gauge fields as follows [53]

$$J := \partial_t w + \partial_k w_k, \quad \text{(A9)}$$
$$J_i := \partial_k \tau_{ik} - \frac{1}{2}\partial_i \tau_{kk} + \partial_t w. \quad \text{(A10)}$$

The harmonic gauge conditions $g^{\alpha\beta}\Gamma^\sigma_{\alpha\beta} = \mathcal{O}(c^{-5}, c^{-4})$ corresponds to $J = \mathcal{O}(c^{-1})$ and $J_i = \mathcal{O}(c^{-1})$. From Eq. (A5), one can see that the difference of the field τ_{ij} with respect to general rerlativity is isotropic. Let us write $\tau_{ij} = \tau_{ij}^{GR} + \chi\delta_{ij}/2$, one has

Solar system phenomenology

The only change is the Shapiro delay at the c^{-4} level

$$c(t_r - t_e)_{ER} = R + \sum_A (1+\gamma_A)\frac{GM_A}{c^2}\ln\left(\frac{\boldsymbol{n}\cdot\boldsymbol{r}_{rA} + r_{rA}}{\boldsymbol{n}\cdot\boldsymbol{r}_{eA} + r_{eA}}\right) + c(t_r - t_e)^{(4)}_{GR}, \quad (14)$$

where

$$\gamma_A = 1 - \frac{1}{2}\frac{M_A^P}{M_A}, \quad (15)$$

and $c(t_r - t_e)^{(4)}_{GR}$ are the remaining c^{-4} terms that are the same as in general relativity.

$$M_A := 4\pi \int_A r^2\ \rho(r) dr, \qquad M_A^P := 4\pi \int_A \frac{r^2\ P(r)}{c^2} dr \quad (16)$$

Solar system phenomenology

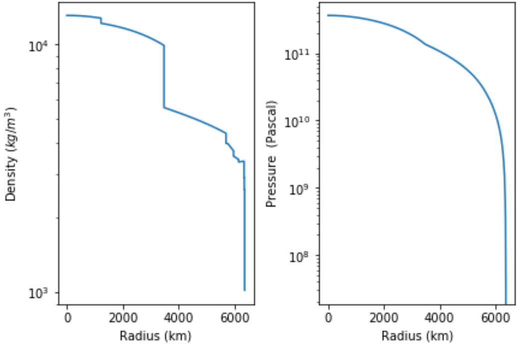

Figure: Energy density and pressure profiles of the Earth (PREM model). Profiles kindly provided by Yanick Ricard from ENS Lyon.

$$\boxed{1 - \gamma_{\oplus} \approx 0.6 \times 10^{-10}} \tag{17}$$

Solar system phenomenology

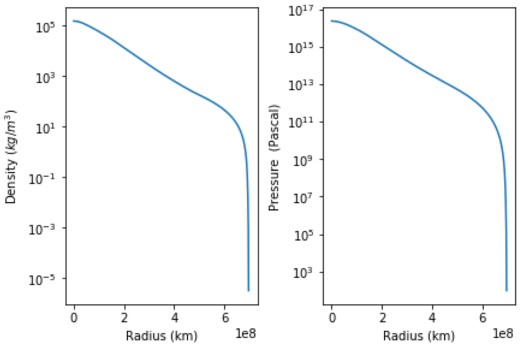

Figure: Energy density and pressure profiles of the Sun (model S).

$$\boxed{1-\gamma_{\odot} \approx 0.5 \times 10^{-6}} \quad (18)$$

Solar system phenomenology

Prediction of Entangled Relativity:

$$\boxed{1 - \gamma_{\odot} \approx 0.5 \times 10^{-6}} \tag{19}$$

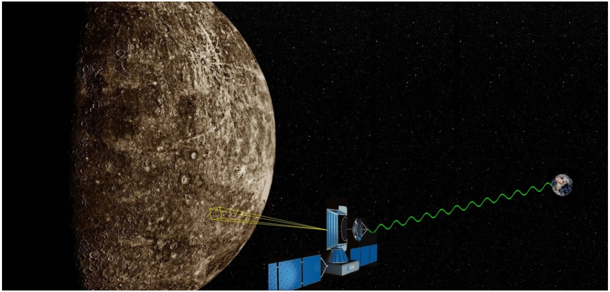

Figure: MORE experiment on BepiColombo targets $\Delta\gamma \pm 6 \cdot 10^{-6}$. 2201.05092

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The standard QFT limit

Path integral formulation (gravity neglected)

Both g and κ are considered to be constant.

$$Z_{\mathrm{ER}} \approx \int \prod_i [\mathcal{D}f_i] \exp\left[\frac{i}{\kappa\epsilon^2}\int d^4x \mathcal{L}_m(f)\right] \tag{20}$$

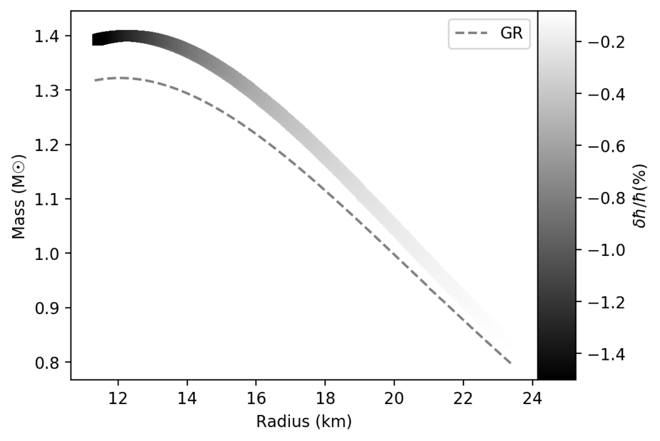
$$c\hbar := \kappa\epsilon^2 \Rightarrow \epsilon =: \sqrt{\frac{c\hbar}{\kappa}}: \text{ Planck energy}$$

Interpretations

- Standard QFT is a specific semi-classical limit ($(g,\kappa) \approx$ constants) of entangled relativity. (Note that $(g,\kappa) \approx$ constants is a consequence of the classical field equations).
- The only parameter of entangled relativity is the Plank energy.
- $\kappa = 8\pi G/c^4 \Rightarrow \boxed{\hbar \propto G}$ such that $\boxed{G \to 0 \Leftrightarrow \hbar \to 0}$.

Neutron stars
Toward experimental tests

A few percent variation of $\kappa \propto \hbar$ (not published yet)



Entangled relativity: conclusion

More parsimonious than Einstein's theory !

$$\frac{1}{c\hbar}\left(\frac{R}{2\kappa}+\mathcal{L}_m\right) \rightarrow -\frac{1}{2\epsilon^2}\frac{\mathcal{L}_m^2}{R} \tag{21}$$

Which recovers general relativity and standard QFT in a generic limit!

& has no elementary length and time scales

& no free parameter at all!

2103.05313 & 2206.03824

$$\boxed{1-\gamma_{\odot} \approx 0.5 \times 10^{-6}} \qquad \text{(unpublished result)}$$

Everyone is welcome to help me out studying this theory! :)
I have very little time to devote to it on my own ...

Additionnal slides

General relativity's issues

inertia can be defined *ex nihilo* in general relativity: violation of Mach's principle

Einstein beleived in the *relativity of inertia*

"c. Mach's Principle. [Spacetime] is completely determined by [matter] [...]. With (c), according to the field equations of gravitation, **there can be no [spacetime] without matter**. " Einstein [1918a]

To the press during his first visit in the US in 1921:

"It was formerly believed that if all material things disappeared out of the universe, time and space would be left. According to relativity theory, however, time and space disappear together with the things." Robinson [2018]

The actual reason for the cosmological contant

The cosmological constant was meant (but failed) to satisfy Mach's Principle of Einstein

Response to the paper of de Sitter. Einstein [1918b]

"If the de Sitter solution were valid everywhere, it would show that the introduction of the λ-term does not fulfill the purpose I intended. Because, in my opinion, the general theory of relativity is a satisfying system only if it shows that the physical qualities of space are *completely* determined by matter alone. Therefore no $g_{\mu\nu}$-field must exist (that is, no space-time continuum is possible) without matter that generates it."

Solar system phenomenology

Non-convervation of stress-energy tensor

$$\nabla_\sigma T^{\mu\sigma} = -\left(\mathcal{L}_m g^{\mu\sigma} - T^{\mu\sigma}\right)\frac{\partial_\sigma \kappa}{\kappa}. \tag{22}$$

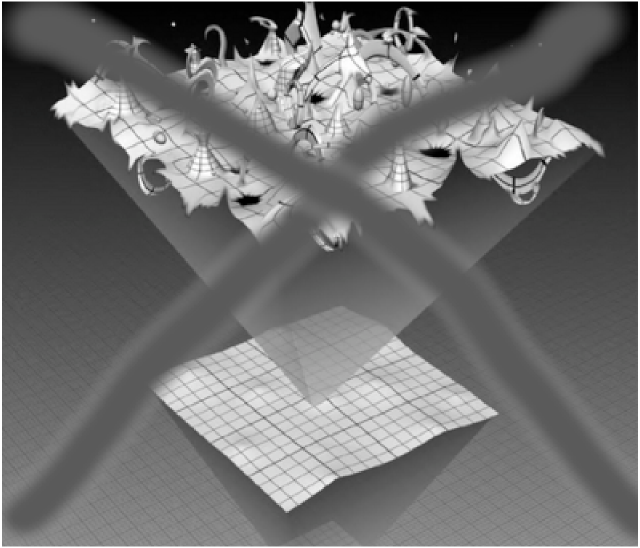
$$U^\sigma \nabla_\sigma U^\mu = \left(g^{\mu\sigma} + U^\mu U^\sigma\right)\frac{\partial_\sigma \kappa}{\kappa}. \tag{23}$$

Same trajectories for test particles as in GR!

$$\begin{aligned}\frac{\mathrm{d}^2 x^i}{\mathrm{d}t} &= a^i_{\mathrm{GR}} + c^{-2}\left\{\partial_i \delta w + \frac{\partial_i \delta\kappa}{\bar{\kappa}}\right\} + \mathcal{O}(c^{-4}) \\ &= a^i_{GR} + \mathcal{O}(c^{-4}),\end{aligned} \tag{24}$$

Just as in 1308.2770 Minazzoli and Hees [2013]

Entangled relativity: no Planck time and length



Origins

Class of theories with intrinsic decoupling found with Aurélien

$$S = \frac{1}{c}\int \mathrm{d}^4_g x \left[\frac{1}{2\alpha}\left(\Phi R - \frac{\omega(\Phi)}{\Phi}(\partial_\sigma \Phi)^2\right) + \sqrt{\Phi}\mathcal{L}_m\right] \qquad (25)$$

$$R^{\mu\nu} = \alpha\frac{1}{\sqrt{\Phi}}\left[T^{\mu\nu} - \frac{1}{2}g^{\mu\nu}T\right] + \frac{1}{\Phi}\left[\nabla^\mu\partial^\nu\Phi + \frac{1}{2}g^{\mu\nu}\Box\Phi\right] + \frac{\omega(\Phi)}{\Phi^2}\partial^\mu\Phi\partial^\nu\Phi, \qquad (26)$$

and

$$\frac{2\omega(\Phi)+3}{\Phi}\Box\Phi + \frac{\omega_\Phi(\Phi)}{\Phi}(\partial_\sigma\Phi)^2 = \alpha\frac{1}{\sqrt{\Phi}}\left[T - \mathcal{L}_m\right] \qquad (27)$$

Entangled relativity corresponds to $\omega(\Phi) = 0$.
$(\gamma = \beta = 1 \;\; \forall \;\; \omega \neq -3/2)$

An ambiguity in the field equations

One needs to know the value of the on-shell matter Lagrangian

$$3\kappa^2 \Box \kappa^{-2} = \kappa \left(T - \mathcal{L}_m \right) \tag{28}$$

- if $\boxed{\mathcal{L}_m = T}$, as argued in e.g. Avelino and Azevedo [2018], then the scalar degree of freedom has no source at all $\Rightarrow$ GR.
- if $\boxed{\mathcal{L}_m = -\rho}$, as argued in e.g. Minazzoli and Harko [2012], then the scalar degree of freedom is sourced by pressure only $\rightarrow$ *Pressuron*. Minazzoli and Hees [2014].
- if $\boxed{\mathcal{L}_m = P}$, one **does not** have general relativity at leading post-Newtonian order. Should only be valid for exotic objects such as *fuzzy dark matter* Arruga et al. [2021]. (Because $\mathcal{L}_m = K - V = P$ for scalar fields).

Post-Newtonian solutions assuming $\nabla_\sigma(\rho_0 u^\sigma) = 0$

No source $\Rightarrow$ GR post-Newtonian phenomenology

$$\mathcal{L}_m = T \qquad \Rightarrow \qquad \Box\phi^2 = 0. \tag{29}$$

Or "Pressuron" $\rightarrow$ name given in Minazzoli and Hees [2014]

$$\mathcal{L}_m = -\rho \qquad \Rightarrow \qquad \frac{1}{\phi^2}\Box\phi^2 = -\frac{\tilde{\kappa}}{\phi}P, \tag{30}$$

Solution for *pressuron* Minazzoli and Hees [2013]

$$\boxed{g^{ER}_{\alpha\beta} = g^{GR}_{\alpha\beta} + \mathcal{O}\left(P/(\rho c^2), 1/c^4\right)} \qquad \Rightarrow PPN : \gamma = \beta = 1 \tag{31}$$

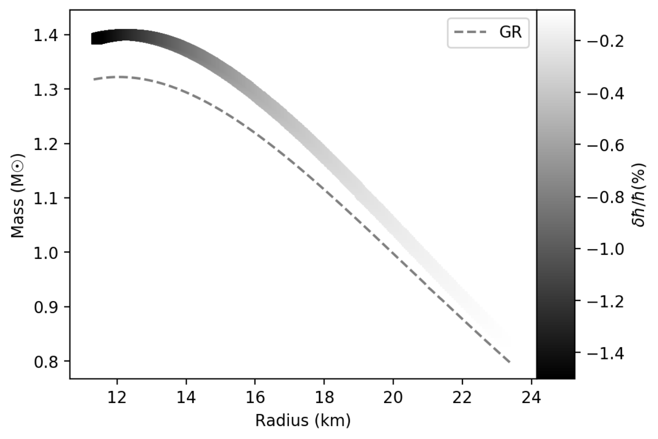
$P/(\rho c^2) = \mathcal{O}(10^{-10})$ for the Earth

However $P/(\rho c^2)$ not negligible for neutron stars

Neutron stars

Toward experimental tests

A few percent variation of $\kappa \propto \hbar$ (not published yet)



Consequences of the intrinsic decoupling

The phenomenology of the theory reduces (or converges) toward the one of general relativity whenever $\mathcal{L}_m \approx T$ on-shell.

- For a universe made of dust and EM radiation, the scalar degree of freedom freezes and one gets GR back at the cosmological level. 2011.14633
- Neutron stars are at max a few percent different from the ones of GR. 2011.14629
- Exterior of (spherical) black holes cannot be distinguished from the ones of GR in astrophysical conditions. 2102.10541
- Gravitationnal waves emmited from the fusion of black-holes are indistinguishables from the ones of GR. 1706.09875

Consequences of the intrinsic decoupling

Broad consequence of the decoupling

κ varies much less than the spacetime metric at the classical level. (Generically, but not always).

Classical limit of entangled relativity ($\mathcal{L}_m \neq \emptyset$)

Quite generically, but not always, one has

$$-\frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\frac{\mathcal{L}_m^2}{R} \equiv \int \mathrm{d}^4x\sqrt{-g}\frac{1}{\kappa}\left(\frac{R}{2\kappa}+\mathcal{L}_m\right) \quad (32)$$

$$\approx \frac{1}{\kappa}\int \mathrm{d}^4x\sqrt{-g}\left(\frac{R}{2\kappa}+\mathcal{L}_m\right) \quad (33)$$

General relativity is a limit of (predicted by) the theory.

Path integral formulation of ER (gravity neglected)

Both g and κ are considered to be constant.

$$Z_{\rm ER} \approx \int \prod_i \mathcal{D}f_i \exp\left[\frac{i}{\kappa\epsilon^2}\int d^4x \mathcal{L}_m(f)\right] \tag{34}$$

Standard QFT on "flat spacetimes" (i.e. gravity neglected)

g is considered to be a constant.

$$Z_{\rm QFT} = \int \prod_i \mathcal{D}f_i \exp\left[\frac{i}{c\hbar}\int d^4x \mathcal{L}_m(f)\right] \tag{35}$$

$c\hbar := \kappa\epsilon^2 \Rightarrow \epsilon =: \sqrt{\frac{c\hbar}{\kappa}}$: Planck energy

Path integral formulation (gravity neglected)

Both g and κ are considered to be constant.

$$Z_{\rm ER} \approx \int \prod_i \mathcal{D}f_i \exp\left[\frac{i}{\kappa\epsilon^2}\int d^4x \mathcal{L}_m(f)\right] \quad (36)$$

$$c\hbar := \kappa\epsilon^2 \Rightarrow \epsilon =: \sqrt{\frac{c\hbar}{\kappa}}: \text{ Planck energy}$$

Interpretations

- Standard QFT is a specific semi-classical limit ($(g, \kappa) \approx$ constants) of entangled relativity. (Note that $(g, \kappa) \approx$ constants is a consequence of the classical field equations).
- The only parameter of entangled relativity is the Plank energy.
- $\kappa = 8\pi G/c^4 \Rightarrow \boxed{\hbar \propto G}$ such that $\boxed{G \to 0 \Leftrightarrow \hbar \to 0}$.

Path integral formulation ($\kappa \approx$ constant only)

Only κ is considered to be constant (as a consequence of the intrinsic decoupling)

$$Z_{\rm ER} \approx \int \mathcal{D}g \prod_i \mathcal{D}f_i \exp\left[\frac{i}{\kappa\epsilon^2}\int d_g^4x \left(\frac{R}{2\kappa} + \mathcal{L}_m\right)\right] \tag{37}$$

Interpretations

- The Core theory of physics can be a limit of entangled relativity when $\kappa \approx$ constant.
- $\mathcal{L}_m$ is unknown at the fundamental level but must be such that it is the standard model of particles when $\kappa = 8\pi G/c^4$, with G being Newton's constant measured on Earth.

Classical limit: gravity

Classical equivalence (provided $\mathcal{L}_m \neq \emptyset$) Ludwig et al. [2015]

$$-\frac{1}{2}\int d^4_g x \frac{\mathcal{L}_m^2}{R} \equiv \frac{1}{\bar{\kappa}}\int d^4_g x \left(\frac{\varphi^2 R}{2\bar{\kappa}} + \varphi \mathcal{L}_m\right) \quad (38)$$

κ: dimensionfull scalar field ⇑ Cauchy well-posed ⇑

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \bar{\kappa}\, T_{\mu\nu} + \varphi^{-2}\left[\nabla_\mu \nabla_\nu - g_{\mu\nu}\Box\right]\varphi^2 \quad (39)$$

$$\nabla_\sigma \left(\varphi T^{\alpha\sigma}\right) = \mathcal{L}_m \nabla^\alpha \varphi \quad (40)$$

Trace metric field equation: $\boxed{3\varphi^{-2}\Box\varphi^2 = \bar{\kappa}\,(T - \mathcal{L}_m)}$ (41)

$$\varphi = -\bar{\kappa}\frac{\mathcal{L}_m}{R} \quad (42)$$

Classical limit: gravity

Classical equivalence (provided $\mathcal{L}_m \neq \emptyset$)

$$-\frac{1}{2}\int \mathrm{d}^4_g x \frac{\mathcal{L}_m^2}{R} \equiv \int \mathrm{d}^4_g x \frac{1}{\kappa}\left(\frac{R}{2\kappa} + \mathcal{L}_m\right) \tag{43}$$

κ: dimensionfull scalar field $\Uparrow$ Cauchy well-posed $\Uparrow$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu} + \kappa^2 \left[\nabla_\mu \nabla_\nu - g_{\mu\nu}\Box\right] \frac{1}{\kappa^2} \tag{44}$$

$$\nabla_\sigma \left(\kappa^{-1} T^{\alpha\sigma}\right) = \mathcal{L}_m \nabla^\alpha \kappa^{-1} \tag{45}$$

Trace metric field equation: $$3\kappa^2 \Box \frac{1}{\kappa^2} = \kappa \left(T - \mathcal{L}_m\right) \tag{46}$$

$$\kappa = -\frac{R}{\mathcal{L}_m} \qquad \left(\kappa = -\frac{R}{T} \text{ in GR}\right) \tag{47}$$

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